Calculations of Soil Pressure on Pipelines in Embankments

Calcul de la poussée des terres sur les conduites dans les remblais

M. V. MALISHEV, *Scientific Research Institute of Foundations and Underground Structures of the U.S.S.R., State Building Committee, Moscow, U.S.S.R.*

SUMMARY

The report sets forth a new method of calculating the pressure of soil on pipelines with a circular cross-section laid in the body of embankments erected with the soil rolled or by hydraulic fill. The method can be used to determine the pressure of the soil on the pipeline, the bending moments, and the lateral and normal forces in any section. Various beds for supporting pipelines are considered. Account is taken of the flexibility of the pipelines, the deformation properties of the soil in the embankment and the bed, seismic action, and pore pressure. For all instances simple equations are given for performing the calculations. The factors and coefficients included in these equations are tabulated. An example of the calculations is given.

SOMMAIRE

L'auteur expose une nouvelle méthode de calcul de la poussée des terres sur les conduites cylindriques mises en place à l'intérieur de remblais hydrauliques ou compactés. La méthode permet de déterminer la poussée des terres sur la conduite, les moments de flexions, les efforts transversaux et normaux dans toute section. On examine différentes conditions d'appui des conduites. On tient compte de la souplesse des conduites, de la propriété de déformation des terres du remblai et de la fondation, de l'influence sismique et de la pression interstitielle. Des équations de calcul simples sont données pour tous les cas, ainsi que les coefficients sous forme de tableaux. On donne aussi un exemple de calcul.

IN PRACTICE three fundamentally different situations determining the selection of the equations for calculating the pressure on pipelines with a round cross-section located in the soil may be encountered: laying in a narrow trench, laying in a wide trench or embankment, and laying by the closed method (Klein, 1957). When laying a pipeline under an embankment the growth of the embankment will lead to the consolidation of a layer of soil equal in height to the diameter of the pipeline and located at each side of it.

Since the compressibility of the pipeline in a vertical direction is usually less than the compression of the layer of soil at the sides of the pipeline, it becomes necessary to take into account the concentration of stresses at the pipeline, as a result of which the load on the pipes will be greater than the total weight of the column of soil lying directly above the pipeline. The proposed method of calculation is also suitable for calculating pipelines laid in wide trenches, the longitudinal section of which does not intersect the section AA′ BB′ indicated in Fig. 1, where ϕ is the angle of internal friction (Malishev, 1958).

At a depth H from the surface the effective vertical pressure of moist soil and soil partly saturated with water will be

$$p_{\text{eff}} = \gamma_{b1} H_w + \gamma_{b2}(H - H_w) - p_{\text{pore}}. \quad (1)$$

The neutral pressure at the same depth will be

$$p_{\text{neutr}} = \gamma_w (H - H_w) + p_{\text{pore}}, \quad (2)$$

where γ_{b1} = bulk density above water level; γ_{b2} = bulk density below water level with account taken of buoyancy; p_{pore} = pore water pressure; γ_w = specific gravity of water. The pore water pressure can be determined on the basis of the available solution of the single-measurement problem relating to the consolidation of a layer of soil increasing in thickness with time (Malishev, 1959; Gibson, 1958).

In order to take into account the condition of equal movements of the pipeline and the soil surrounding it, the

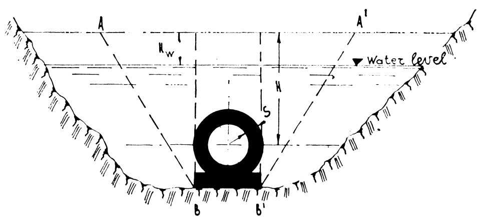

FIG. 1. Section through pipeline laid in a wide trench.

solution of the problem of the deformation of an elastic plate having an annular groove secured with an elastic tunnelling has been employed (Malishev, 1958), as well as other engineering methods that take into account a number of additional factors influencing the pressure of the soil on the pipeline. The basic solution of the problem is as follows:

$$\begin{aligned} \sigma_r &= p_0 + p_2 \cos 2\theta; \\ \tau_{r\theta} &= t_2 \sin 2\theta; \\ M &= (R^2/6)(2p_2 + t_2) \cos 2\theta; \\ N &= (R/3)[3p_0 - (p_2 + 2t_2) \cos 2\theta]; \\ Q &= (R/3)(2p_2 + t_2) \sin 2\theta; \end{aligned} \quad (3)$$

where σ_r and $\tau_{r\theta}$ are respectively the normal radial and tangential stresses on the periphery of the pipeline, M, N, and Q are the bending moment, normal (compression is considered to be positive), and transverse forces therein, R is the average radius of the pipeline, and p_0, p_2, and t_2 are factors determined from the equations:

$$\left. \begin{aligned} p_0 &= \bar{p}'_0 \cdot p_{\text{eff}} + p_{\text{neutr}}; \\ p_2 &= \bar{p}'_2 \cdot p_{\text{eff}}; \\ t_2 &= \bar{t}'_2 \cdot p_{\text{eff}} \end{aligned} \right\} \quad (4)$$

TABLE I. FACTORS $\bar{p}_2$ AND $\bar{t}_2$.

	$\xi_0 = 0.2$		$\xi_0 = 0.3$		$\xi_0 = 0.4$		$\xi_0 = 0.5$		$\xi_0 = 0.6$		$\xi_0 = 0.7$	
α'	$\bar{p}_2$	$\bar{t}_2$	$\bar{p}_2$	$\bar{t}_2$	$\bar{p}_2$	$\bar{t}_2$	$\bar{p}_2$	$\bar{t}_2$	$\bar{p}_2$	$\bar{t}_2$	$\bar{p}_2$	$\bar{t}_2$
0	1.028	−0.012	0.931	−0.007	0.822	0.000	0.700	0.008	0.572	0.018	0.435	0.023
0.05	0.940	0.064	0.851	0.060	0.753	0.060	0.643	0.060	0.526	0.056	0.402	0.053
0.10	0.880	0.112	0.798	0.105	0.705	0.096	0.605	0.090	0.492	0.082	0.378	0.071
0.15	0.832	0.148	0.756	0.140	0.666	0.129	0.573	0.115	0.461	0.102	0.359	0.084
0.20	0.796	0.180	0.721	0.168	0.636	0.153	0.545	0.135	0.448	0.116	0.342	0.095
0.25	0.768	0.204	0.693	0.189	0.612	0.171	0.525	0.150	0.430	0.130	0.329	0.102
0.30	0.740	0.228	0.672	0.210	0.581	0.186	0.505	0.160	0.416	0.138	0.318	0.110
0.35	0.720	0.244	0.651	0.221	0.576	0.198	0.490	0.173	0.402	0.146	0.308	0.114
0.40	0.704	0.256	0.634	0.235	0.558	0.210	0.475	0.183	0.388	0.150	0.299	0.120
1.00	0.587	0.379	0.524	0.325	0.457	0.273	0.388	0.224	0.315	0.176	0.240	0.130

where

$$\left.\begin{aligned} \bar{p}_2' &= \bar{p}_2\left[\frac{1+\alpha_1}{2} + \frac{3}{2}(\alpha_1 - 1)\cdot\frac{\bar{p}_0}{\bar{p}_2 + 2\bar{t}_2}\right]; \\ \bar{t}_2' &= \bar{t}_2\left[\frac{1+\alpha_1}{2} + \frac{3}{2}(\alpha_1 - 1)\cdot\frac{\bar{p}_0}{\bar{p}_2 + 2\bar{t}_2}\right]; \\ \bar{p}_0' &= \bar{p}_0\frac{1+\alpha_1}{2} + \frac{1}{6}(\alpha_1 - 1)(\bar{p}_2 + 2\bar{t}_2). \end{aligned}\right\} \quad (5)$$

The factors $\bar{p}_2$ and $\bar{t}_2$ are taken from Table I depending upon ξ_0, the lateral pressure factor of the soil skeleton, and α', the stiffness factor (Malishev, 1963).

$$\alpha' = 0.97(E_{\text{soil}}/E_{\text{pipe}})(R/S)^3. \quad (6)$$

The factor $\bar{p}_0$ is equal to

$$\bar{p}_0 = \frac{1+\xi_0}{1+\xi_0+\alpha'\left(\dfrac{S}{R}\right)^2}. \quad (7)$$

In these equations E_{soil} and E_{pipe} are respectively the modulus of deformation of the soil and the modulus of elasticity of the pipe material; S is the thickness of the pipe wall. The factor α_1, which takes into account the position of the zero displacement line, will be (Malishev, 1963):

$$\alpha_1 = \frac{1 + \dfrac{E_{\text{soil}}}{E_{\text{bed}}} + A_2\left(1 - \dfrac{E_{\text{soil}}}{E_{\text{bed}}}\right)\dfrac{l}{R + \dfrac{S}{2}}}{A_1 + \dfrac{E_{\text{soil}}}{E_{\text{bed}}}} \quad (8)$$

where $l/(R + S/2)$ is the relative depth of the pipeline (Fig. 2, a and b), A_1 and A_2 are factors determined from Table II, and E_{bed} is the modulus of deformation of the bed.

In the upper half of the pipe the values M, N, and Q are determined by means of Eq 3. The additional values of the moment and the normal force to be added to the above caused by the weight of the pipeline and the weight of the water completely filling its cross-section are set forth in Table III. These equations are valid for the entire pipe circumference.

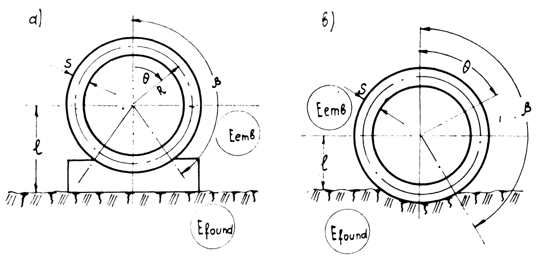

FIG. 2. (a) Section through pipe laid on a concrete bed; (b) section through pipe laid directly on natural ground.

TABLE II. FACTORS A_1, A_2, AND A_3.

ξ_0	α'	A_1	A_2	A_3
0.2	0	1	0.251	0.334
0.2	0.05	1.038	0.255	0.334
0.2	0.1	1.067	0.259	0.334
0.2	0.2	1.134	0.267	0.334
0.2	0.4	1.268	0.284	0.334
0.3	0	1	0.234	0.306
0.3	0.05	1.031	0.238	0.306
0.3	0.1	1.061	0.241	0.306
0.3	0.2	1.122	0.248	0.306
0.3	0.4	1.245	0.263	0.306
0.4	0	1	0.215	0.273
0.4	0.05	1.027	0.218	0.273
0.4	0.1	1.055	0.220	0.273
0.4	0.2	1.109	0.226	0.273
0.4	0.4	1.219	0.238	0.273
0.5	0	1	0.192	0.238
0.5	0.05	1.024	0.195	0.238
0.5	0.1	1.048	0.197	0.238
0.5	0.2	1.095	0.202	0.238
0.5	0.4	1.190	0.211	0.238
0.6	0	1	0.168	0.203
0.6	0.05	1.020	0.170	0.203
0.6	0.1	1.040	0.172	0.203
0.6	0.2	1.081	0.175	0.203
0.6	0.4	1.162	0.182	0.203

TABLE III. ADDITIONAL MOMENT AND NORMAL FORCE DUE TO WEIGHT OF PIPE AND WATER

	Caused by weight of pipe	Caused by weight of water in pipe
Additional moment ΔM	$\gamma_{\text{pipe}}SR^2(1 - \frac{1}{2}\cos\theta - \theta\sin\theta)$	$\gamma_w R^3(\frac{1}{2} + \frac{\theta}{2}\sin\theta - \frac{1}{4}\cos\theta)$
Additional normal force ΔN	$\gamma_{\text{pipe}}SR(\theta\sin\theta - \frac{1}{2}\cos\theta)$	$-\gamma_w R^2(1 - \frac{1}{4}\cos\theta - \frac{\theta}{2}\sin\theta)$

When the pipeline is installed on a concrete bed (Fig. 2a), and considering that there are only two points of contact on the section where the pipeline adjoins the foundation, *viz.* $\theta = \beta$ and $\theta = 2\pi - \beta$, we obtain the bending moments in the lower half of the pipe circumference in a section $\beta \geqslant \theta \geqslant \pi/2$ equal to

$$M = K_1 T_1 R, \quad (9)$$

TABLE IV. FACTORS K_1, K_2, K_3, K_4, AND K_5

	$E_{red}/E_{bed} = 0.9$		$E_{red}/E_{bed} = 0.8$		$E_{red}/E_{bed} = 0.6$		$E_{red}/E_{bed} = 0.4$		$E_{red}/E_{bed} = 0.1$				
β^0	K_2	K_3	K_2	K_3	K_2	K_3	K_2	K_3	K_2	K_3	K_1	K_4	K_5
180	—	—	—	—	—	—	—	—	—	—	0.6366	0.6366	0.6366
170	0.0057	0.0044	0.0126	0.0096	0.0314	0.0240	0.0625	0.0478	0.1846	0.1411	0.4726	0.5530	0.4666
160	0.0083	0.0042	0.0179	0.0091	0.0423	0.0214	0.0772	0.0391	0.1719	0.0870	0.3322	0.4778	0.3067
150	0.0086	0.0020	0.0182	0.0043	0.0409	0.0096	0.0700	0.0165	0.1337	0.0314	0.2180	0.4134	0.1634
140	0.0073	−0.0003	0.0152	−0.0007	0.0330	−0.0015	0.0540	−0.0237	0.0938	−0.0041	0.1305	0.3600	0.0386
130	0.0053	−0.0017	0.0109	−0.0035	0.0230	−0.0073	0.0364	−0.0116	0.0594	−0.0189	0.0688	0.3180	−0.0650
120	0.0032	−0.0019	0.0065	−0.0038	0.0134	−0.0078	0.0207	−0.0120	0.0325	−0.0188	0.0296	0.2867	−0.1463
110	0.0015	−0.0012	0.0030	−0.0024	0.0061	−0.0048	0.0092	−0.0073	0.0141	−0.0112	0.0088	0.2657	−0.2041
100	0.0004	−0.0004	0.0007	−0.0007	0.0015	−0.0015	0.0022	−0.0022	0.0034	−0.0034	0.0012	0.2538	−0.2386
90	0	0	0	0	0	0	0	0	0	0	0	0.2500	−0.2500

where

$$T_1 = R[(p_2 + 2t_2)/3 + \bar{p}'_0 p_{eff}]. \tag{10}$$

The factor K_1 is determined from Table IV.

In the case of normally supporting a foundationless pipeline (Fig. 2b) on soft soil (sand, clay), the compression moduli of which are E_{bed} and E_{soil}, and presuming that the reactive pressures are distributed uniformly along the contact with the bed, the values of the bending moments in the lower part of the pipeline with $\theta = \pi$ will be

$$M = R^2\left[\frac{2p_2 + t_2}{6} + K_2\frac{T_1}{R}\right] \tag{11}$$

and with $\theta = \beta$, and $\theta = 2\pi - \beta$

$$M_{\theta=\beta} = R^2\left[\frac{2p_2 + t_2}{6}\cos 2\beta + K_3\frac{T_1}{R}\right]. \tag{12}$$

The factors K_1 and K_2 are taken from Table IV, the force T_1 is calculated by means of Eq 10, p_2 and t_2 from Eq 4. The reduced modulus of deformation E_{red}, determined from the condition of equality of displacements at a distance from the pipeline equal to R/A_3, where the additional stresses are practically negligible, is calculated as follows:

$$E_{red} = E_{soil} \cdot E_{bed}/[A_3(E_{bed} - E_{soil}) + E_{soil}]. \tag{13}$$

The factor A_3 is contained in Table I.

If in the case of normal bearing on soft soil E_{bed} is approximately equal to E_{soil}, then the calculation for the lower half of the pipe circumference is carried out in the same way as for the upper one, according to Eq 3.

When a foundationless pipeline is supported on half-rock and rock the moment in the section $\theta = \pi$ is determined according to the equation

$$M = K_4 T_1 R, \tag{14}$$

and in the section $\theta = \beta$ and $\theta = 2\pi - \beta$ by

$$M = K_5 T_1 R. \tag{15}$$

The values of K_4 and K_5 are contained in Table IV, and the value of T_1 is determined, as previously, from Eq 10.

When designing pipelines for seismic regions, the rated values of the maximum bending moments are increased, and to the moments calculated by means of Eq 3 there should be added moments equal to

$$\Delta M_{seism} = (R^2/6)(p_0 + p_2)K_6 \tag{16}$$

where all the previous designations have been used, and the factor $K_6 = 0.025$ for a rated seismicity of 6 points, 0.05 for 7 points, and 0.1 for 8 points.

For estimating the influence of the difference in the deformation of the pipeline and the adjacent soil on the redistribution of the vertical stresses a factor K_{emb} can be employed, determined as follows:

$$K_{emb} = \frac{p_0 + \frac{1}{3}(p_2 + 2t_2)}{p_{eff} + p_{neutr}}. \tag{17}$$

It should be noted that in the proposed method of calculation the solution from the theory of elasticity for an infinite plate has been employed. When using this solution, however, for a semi-infinite plate the error resulting from the substitution of an infinite plate for a semi-infinite one if $H \geqslant 5R$ does not exceed 5 per cent in the stresses at the pipeline, diminishing with an increase in the ratio H/R. This circumstance, noted by Malishev (1963), permits the use of the equations set forth above for calculating pipelines laid in embankments in many important instances, since it is very difficult to obtain an elastic-plastic solution.

EXAMPLE OF CALCULATIONS

A reinforced concrete pipeline has an average diameter of 1.8 m and a wall thickness of 20 cm, and is laid on soft soil with a specially prepared longitudinal section. The embankment is to be laid of fine sand soil and its height above the centre line of the pipe H will be 50 m. The level of the water in the embankment will be below the surface at a distance of $H_w = 20$ m from the top. The void ratio of the sand $\epsilon = 0.7$, the specific gravity 2.65 tons/cu.m., the gravimetric sand humidity above water level $W = 0.1$. The modulus of deformation of the soil $E_{soil} = 120$ kg/sq.cm., of the bed $E_{bed} = 400$ kg/sq.cm., of the reinforced concrete $E_{pipe} = 165{,}000$ kg/sq.cm. The lateral pressure factor of the soil skeleton $\xi_0 = 0.37$. The pipeline is laid in the bed to a depth of 30 cm ($l = 0.9 + 0.2/2 - 0.3 = 0.7$ m).

The bulk density above water level will be

$$\gamma_{b1} = [(1 + 0.1) \times 2.65]/(1 + 0.7) = 1.72 \text{ tons/cu.m.},$$

while the bulk density below water level with account taken of buoyancy will be

$$\gamma_{b2} = (2.65 - 1)/(1 + 0.7) = 0.97 \text{ tons/cu.m.}$$

By substitution in Eq 1 disregarding the pore water pressure to provide a safety factor, we obtain

$$p_{eff} = 0.72 \times 20 + 0.97(50 - 20) = 63.5 \text{ tons/sq.m.},$$

and from Eq 2

$$p_{neutr} = 1.0 \times (50 - 20) = 30 \text{ tons/sq.m.}$$

Substituting in Eq 6 to calculate the stiffness factor, we have

$$\alpha' = 0.97 \frac{120}{165000} \left(\frac{0.9}{0.2}\right)^3 = 0.064$$

The value of $\bar{p}_0$ is determined from Eq 7

$$\bar{p}_0 = \frac{1 + 0.37}{1 + 0.37 + 0.064\left(\frac{0.2}{0.9}\right)^2} = 0.997.$$

From Table I, employing linear interpolation, we obtain $\bar{p}_2 = 0.772$ and $\bar{t}_2 = 0.070$, and from Table II, for the instance under consideration $A_1 = 1.044$, $A_2 = 0.224$, and $A_3 = 0.283$. Whence from Eq 8:

$$\alpha_1 = \frac{1 + \frac{120}{400} + 0.224\left(1 - \frac{120}{400}\right)\frac{0.7}{1.0}}{1.044 + \frac{120}{400}} = 1.05.$$

By means of Eq 5 the following values are calculated $\bar{p}'_2 = 0.855$, $\bar{t}'_2 = 0.077$, $\bar{p}'_0 = 1.028$ and from Eq 4: $p_0 = 1.028 \times 63.5 + 30 = 95.2$ tons/sq.m., $p_2 = 0.855 \times 63.5 = 54.4$ tons/sq.m., and $t_2 = 0.077 \times 63.5 = 4.9$ tons/sq.m.

Thus for the upper half of the pipe circumference we have

$$\sigma_r = 95.2 + 54.4 \cos 2\theta \text{ tons/sq.m.};$$

$$\tau_{r\theta} = 4.9 \sin 2\theta \text{ tons/sq.m.}$$

$$M = (0.9^2/6)(2 \times 54.4 + 4.9) \cos 2\theta = 15.35 \cos 2\theta \text{ ton-meters/m.}$$

$$N = (0.9/3)[3 \times 95.2 - (54.4 + 2 \times 4.9) \cos 2\theta] = 85.6 - 19.3 \cos 2\theta \text{ tons/m.}$$

For calculating the moments in the lower half of the pipe circumference from Eq 10 we obtain

$$T_1 = 0.9[(54.4 + 2 \times 4.9)/3 + 1.028 \times 63.5] = 78 \text{ tons/m.}$$

The angle $\beta = \pi - \arc\cos 1/(R + S/2)$, whence $\beta = 135°$.

Substituting in Eq 13 yields

$$E_{red} = 120 \times 400/[0.283(400 - 120) + 120] = 240 \text{ kg./sq.cm.}$$

and $E_{red}/E_{bed} = 240/400 = 0.6$, and from Table IV we find that, with $\beta = 135°$, the factors $K_2 = 0.0280$ and $K_3 = -0.0044$. From Eq 11

$$M_{\theta=180°} = 0.9^2[\tfrac{1}{6}(2 \times 54.4 + 4.9) + 0.028(78/0.9)] = 15.9 \text{ ton-meters/m.}$$

and from Eq 12 for $\cos 270° = 0$.

$$M_{\theta=135°} = 0.9^2[-0.0044(78/0.9)] = -0.4 \text{ ton-meters/m.}$$

The additional moments caused by the weight of the pipe itself and of the water therein are negligible and with $\theta = 0$ are respectively $2.65 \times 0.2 \times 0.9^2 \times 0.5 = 0.2$ ton meters/m and $1 \times 0.9^3 \times 0.25 = 0.2$ ton meters/m. The values of the moments and the normal forces so obtained enable the fittings of the pipeline to be calculated. The factor K_{emb}, showing how the pressure on the pipeline increases in comparison with the weight of the column of soil above it, is calculated by means of Eq 17:

$$K_{emb} = [95.2 + \tfrac{1}{3}(54.4 + 2 \times 4.9)]/(63.5 + 30) = 1.25,$$

i.e., the pressure is increased by 25 per cent.

REFERENCES

GIBSON, R. E. (1958). The progress of consolidation in a clay layer, increasing in thickness with time. *Géotechnique*, No. 4.

KLEIN, G. K. (1957). *Calculation of pipes laid in ground.* Gosstroyizdat Publishing House.

MALISHEV, M. V. (1958). On the pressure of hydraulic fill on the pipeline. *Proc. Brussels Conference on Earth Pressure Problems*, Vol. 1.

——— (1959). Consolidation of water-saturated soil with gradual increase of layer thickness. *Osnovaniya, fundamenti i mekhanika gruntov*, No. 3.

——— (1963). Methods of statical calculation of collecting mains and pipelines laid under high embankments. *All-Union Scientific Research Institute VODGEO*, Collected Articles of Institute, Issue 4, VODGEO Publishing House.